Refer the pseudocode.
We want to count the # of times the compare happens inside the for loop (line 4).
All other lines happen in constant time

The difference in the analysis that we will do for this problem is INSTEAD of counting the # of compares done in ONE CALL to PARTITION (and then multiplying by # of times it is called)
We instead determine the # of compares TOTAL over all calls to PARTITION.

So RANDOMIZED QUICKSORT EXPECTED RUNTIME

X is a R.V. for # of COMPARES (# of times line 4 executes)
We want $E[X]$ which is EXPECTED # of compares.

Let's label the elements of the array z_i so $A = \{z_1, z_2, \ldots, z_n\}$

Assume (without loss of generality) all z's are unique and $Z_{ij} = \{Z_i, Z_{i+1}, \ldots, Z_j\}$ = set of Z's btwn i & j inclusive.

How many times (@ most) can 2 particular elements be compared to each other?

↳ NOTE: comparisons take place btwn a PIVOT & a NON-PIVOT

Once a PIVOT is compared to all elements of its group it gets put in its rightful place & is NEVER COMPARED (to anything else) AGAIN.

→ So ⇒ AT MOST ONCE (CAN BE COMPARED $\emptyset$ TIMES).

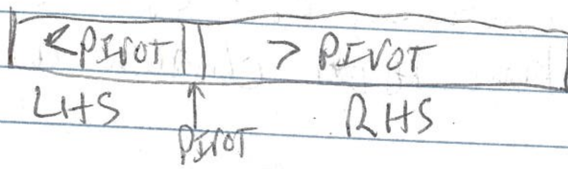
Let's define Ind. R.V.'s X_{ij}

$$X_{ij} = \begin{cases} 1 & \text{if } Z_i \text{ is compared to } Z_j \\ 0 & \text{otherwise.} \end{cases}$$

If we add up all the X_{ij}'s we get the $E[X]$. Note many will be 0 & some will be 1.

During a PARTITION CALL

After 1 call to PARTITION



For:
10.4.2021

1. Which are guaranteed to never be compared to each other
2. Which MAY or MAY NOT be compared to each other?
   └ when will they, or when aren't they?

if either becomes a PIVOT | all other times.

What are all the possible X_{ij}'s?
Since 2 elements are compared @ most once to each other
we force $i < j$ so.

$$\cancel{X} \quad X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}$$

$$E[X] = E\left[\sum\sum X_{ij}\right] = \sum\sum E[X_{ij}] = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} P(z_i \text{ is compared to } z_j)$$

$P(z_i$ is comp. to $z_j) = P(z_i$ OR z_j is chosen as 1st PIVOT in the set$)$

$= P(z_i$ is chosen as 1st pivot$)$
$+$
$P(z_j$ is " " " "$)$

$= ?$

Consider subarray Z_{ij} containing z_i & z_j $\{z_i, z_{i+1}, \dots, z_j\}$
what chance does z_i have to be chosen as a pivot

$$\frac{1}{\text{size of subarray}}$$

$= \frac{1}{j-i+1}$ SAME for Z_j to be chosen as 1st pivot

So:

$P(Z_i \text{ is compared to } Z_j) = \frac{1}{j-i+1} + \frac{1}{j-i+1} = \frac{2}{j-i+1}$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{2}{j-i+1} = 2\sum\sum \frac{1}{j-i+1}$$

change denominator by setting $\underline{k = j-i}$.

$$= 2\sum_{i=1}^{n-1}\sum_{k=1}^{n-i} \frac{1}{k+1}$$

and b/c $\frac{1}{k+1} < \frac{1}{k}$

$$< 2 \underbrace{\sum_{i=1}^{n-1}}_{(n-1)} \underbrace{\sum_{k=1}^{n-i} \frac{1}{k}}_{\ln(n)+\Theta(1)}$$

$$2 \quad (n-1) \quad \ln(n)+\Theta(1)$$

$= 2(n-1)\cdot(\ln(n) + \Theta(1)) = O(n \lg n)$

UPPER BOUND on EXPECTED RUNTIME.

AND since we already determined best case run time
was $\Theta(n \lg n)$

the EXPECTED R.T. $= \Theta(n \lg n)$. as well

since expected can't be better than best