CS 305
Design and Analysis of Algorithms

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Instructor: Michael Eckmann

Today's Topics

- Questions/Comments?
- Recall Mergesort
- Recurrence relations for divide and conquer algorithms
- Recursion trees

MergeSort pseudocode
see code in section 2.3 of CLRS

Analyzing recursive algs

- How did we analyze mergesort for runtime last class?

Analyzing recursive algs

- How did we analyze mergesort for runtime last class?
- We used the recursion tree method

Recurrence Relations

- For MergeSort
- Time(n) = 2 * Time(n/2) + n
  - Number of recursive calls is 2
  - The time for one recursive call is Time(n/2)
  - The time during one call to MergeSort is n
- In general for divide and conquer
- T(n) = a * T(n/b) + f(n)
  - Number of recursive calls is a
  - n/b is size of list in a recursive call
  - The time in a call is f(n)

Recurrence Relations

- In other words, a divide and conquer algorithm that creates *a* subproblems each a factor of *1/b* the size of the original problem and takes *f(n)* amount of time to do the divide and combine steps.
- T(n) = a * T(n/b) + f(n)
    - Number of recursive calls is a
    - n/b is size of list in a recursive call
    - The time in a call is f(n)

Recurrence Relations

- Let me draw the recursion tree for arbitrary a and b.
- How many leaves?
- Let's see an example with explicit values for a and b.
- How many leaves?
- $T(n) = a * T(n/b) + f(n)$
- Let's prove, by induction, that the number of leaves in a perfect binary tree is 2^d, where d = depth of the tree.

Logarithms

$y = \log_b a$ iff $b^y = a$, $a > 0$, $b > 0$ and $b \neq 1$

- In English: one way to read $\log_b a$ is "what exponent of b results in a?"
- When b=2, $\log_2 a$ is the number of times a can be cut in half (until you hit 1.)
- $\log_2 1024 = ?$ It is the power of 2 that gives you 1024 (or the number of times 1024 be cut in half until you hit 1.)

Logarithms

- Let's show some properties of logarithms on the board.