I WILL tell us the amortized cost of 1 of m operations, (but not prove)
n of which are MAKE-SETS.

~~NOTE~~: Before I do I need to introduce an interesting ~~family of functions~~ & things

(1) meaning of $f^{(a)}(x)$: $\quad f^2(x) = f(f(x))$

$$f^4(x) = f(f(f(f(x))))$$

this is called functional iteration (see p. 58 in text)

(2) Define a function

$$A_k(j) = \begin{cases} j+1 & \text{if } k=0 \\ A_{k-1}^{(j+1)}(j) & \text{if } k \geq 1 \end{cases}$$

Let's get a feel for this

$$A_1(1) = A_0^{(2)}(1) = A_0(A_0(1)) = A_0(2) = 3$$

$$A_2(1) = A_1^{(2)}(1) = A_1(A_1(1)) = A_1(A_0^{(2)}(1)) = A_1(3)$$

$$= A_0^{(4)}(3) = \underbrace{A_0(\underbrace{A_0(\underbrace{A_0(A_0(3))}_{4})}_{5})} = 7$$

$$A_3(1) = A_2^{(2)}(1) = A_2(A_2(1)) \overset{6}{=} A_2(7) = A_1^{(8)}(7)$$

$$= A_1(A_1(A_1(A_1(A_1(A_1(A_1(A_1(7))))))))$$

$$= \qquad \underbrace{A_0^{(8)}(7)}_{15}$$

180

side

$x_2(2) =$

$A_1^{(3)}(2)$

$= A_1(A_1(A_1(2)))$

$A_0^{(6)}(5)$

$A_0^{(12)}(11)$

$= 23$

$\underbrace{\quad\quad}_{15}$ 1

$\underbrace{\quad\quad\quad}_{31}$ 2

$\underbrace{\quad\quad\quad\quad}_{63}$ 3

4

5

6

7

8

$16-1$ 1

$32-1$ 2

$64-1$ 3

$2^{11}-1$ 8

$$A_3(1) = 2^{11} - 1 = 2047$$

$$A_4(1) = A_3^{(2)}(1) = A_3(A_3(1)) = A_3(2047) = A_2^{(2048)}(2047)$$

$$= A_2(\ldots A_2(2047)) = A_2^{(2048)}(2047)$$

~~$\gg 2^{2048} \gg 10^{80}$~~

There's a closed form for $A_2(j)$ in text which $= 2^{j+1}(j+1) - 1$

So: clearly that is $\gg A_2(2047) > 2^{2048} \gg 10^{80}$

which is the estimated # of atoms in the observable universe

So can we agree that $A_k(j)$ grows quickly as k increases?

Now we define the inverse of that $A_k(j)$ function

$$\alpha(n) = \min\{k : A_k(1) \geq n\}$$

that is $\alpha(n)$ is the smallest $k \ni A_k(1) \geq n$

$\alpha(2) = \min\{k : A_k(1) \geq 2\} = 0$, b/c $A_0(1) = 2$

$\alpha(3) = 1$, $A_1(1) = 3$ which is ≥ 3.

$\alpha(4) = 2$, b/c $A_2(1) = 7$, but $A_1(1) = 3$.

$\alpha(1024) = 3$ b/c $A_3(1) = 2047 \geq 1024$.

$\alpha(2047) = 3$

$\alpha(2048) = 4$

$\alpha(?) > 4$

$\alpha(n) \leq 4$ for all practical purposes

This inverse function α grows incredibly slowly.

A seq. of m operations, n of which are MAKESETs

takes $O(m\,\alpha(n))$ worst case time (proof in text).

& for all ~~practical~~ conceivable disjoint set problems $\alpha(n) \leq 4$

So can be thought of as a constant. effectively $\Theta(m)$ time

If UNION on equal rank roots,
increment rank of new root.

FIND-SET $\Rightarrow$ no ranks updated
MAKE-SET $\Rightarrow$ rank $\leftarrow 0$.

~~now show pg. 178, 179, 180, 181~~

Next Topic Graph Algorithms (the connected components was a good lead in to this).

$$G = (V, E)$$

Given $|V|$ vertices, how many edges can a graph have?
(note: excluding multigraphs which allow multiple edges btwn two V's) edge to itself is allowed though.

how many V's for the 1^{ST} V of the edge? $|V|$
" 2^{ND} " ? $|V|$

$|V|^2$ so $|E| = O(|V|^2)$

therefore $|E| \leq |V|^2$.

If graph is connected, what's the minimum # of edges?

$$|E| \geq |V| - 1.$$

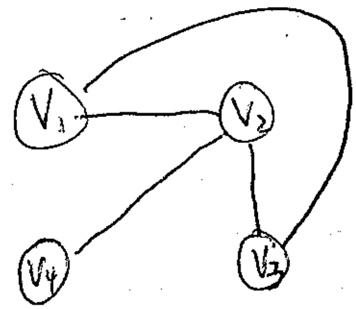
a DENSE graph $\Rightarrow$ $|E| \approx |V|^2$.
a SPARSE " $\Rightarrow$ $|E| \ll |V|^2$.

You told me there were 2 ways to represent the edges

ADJACENCY MATRIX.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}$$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$



Notice symmetric about the main diagonal, if directed, not neccessarily sym.

1's can be replaced by weights for a weighted graph.

ADJACENCY LIST

V_1 : $V_2 \rightarrow V_3$
V_2 : $V_1 \rightarrow V_3 \rightarrow V_4$
V_3 : $V_2 \rightarrow V_1$
V_4 : V_2

SPACE requirement for ADJ. LIST :

$$\Theta(|V| + |E|)$$