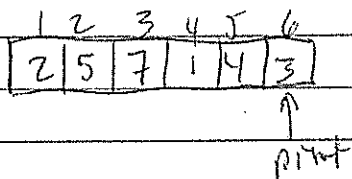


CH 7 QUICKSORT
 CH. 5 & APPROX C.

Problem w/ MERGESORT is SPACE is $\Theta(n)$.

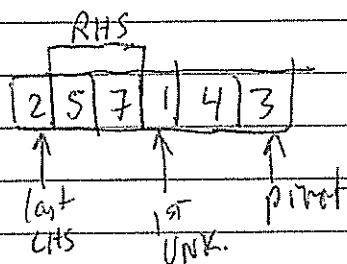
QUICKSORT idea is choose a pivot & place all $<$ pivot on LHS & all \geq pivot on RHS.
 PIVOT will be in final place. then do same to each side

EX: suppose we use last element as pivot:

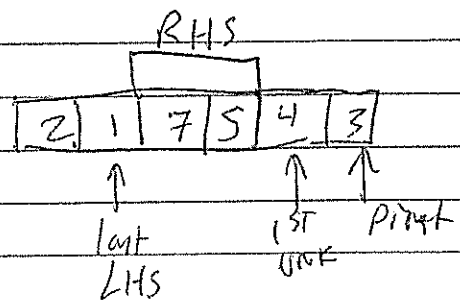


keep index of last of LHS & 1st UNK.

2 < 3 so last of LHS is ↓.
 5 > 3
 7 > 3
 so

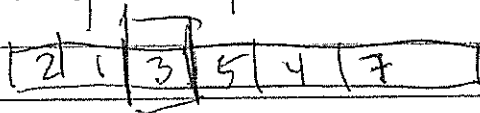


1 < 3 swap with 5. \Rightarrow



4 > 3 so it stays on RHS.

finally swap 3 w/ last LHS + 1 index



~~QUESTION~~
~~QUESTION~~

this is called partition & it is done in place
then do partition on each side (like Merge Sort).

What does the PARTITION FUNCTION NEED AS PARAMS.?

LIST, START & END INDEXES

WHAT OTHER VARS WILL IT NEED?

PIVOT, LASTLHS, FIRSTUNK.

WILL BE CALLED LIKE:

PIVOT ← PARTITION(LIST, START, END)

PARTITION(LIST, START, END)

1. PIVOT ← END
2. LASTLHS ← START - 1
3. FIRSTUNK ← START
4. while FIRSTUNK < END
5. if LIST[FIRSTUNK] < LIST[PIVOT]
6. LASTLHS++
7. SWAP(LIST, FIRSTUNK, LASTLHS)
8. FIRSTUNK++
9. SWAP(LIST, PIVOT, LASTLHS+1)
10. RETURN. LASTLHS+1.

QUICK SORT (LIST, START, END)

1. IF START < END
2. PIVOT ← PARTITION(LIST, START, END)
3. QUICKSORT(LIST, START, PIVOT - 1)
4. " " (" " PIVOT + 1, END)

When we have an algorithm what do we usually like to analyze?
Space &
Runtime

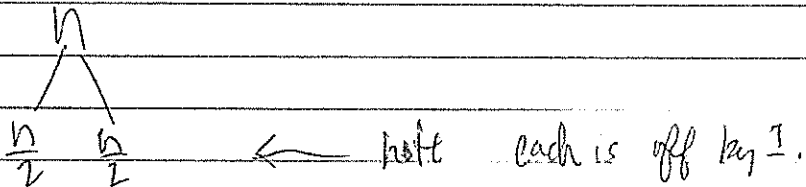
└ Best Case
└ Worst Case

How LONG DOES PARTITION TAKE?

loop iterates n times where $n = \text{END} - \text{START} + 1$
 $\Rightarrow \Theta(n)$

For Best Case

do fewest calls to ~~SS~~ SS. \Rightarrow cent in HALF all the time.



| | | |

this should look familiar.

$$\text{BEST CASE: } T(n) = 2T\left(\frac{n}{2}\right) + \Theta(n)$$

$$a=2, b=2 \quad f(n) = \Theta(n) \quad n^{\log_b a} = n = \Theta(n)$$

$$\text{CASE 2: } T(n) = \Theta(n \lg n)$$

~~START HERE~~

this is now a randomized algorithm.

are we closer to $n \lg n$ or n^2 ?

NEED to know some probability concepts. to answer this.

PROBABILITY

Flip a coin - if both sides equally likely? \Rightarrow

Likelihood of HEADS = 50%

If flip it a million times expect 50% heads

Def'n's:

SAMPLE SPACE S is set of all possible outcomes

e.g. $S_{\text{coin}} = \{H, T\}$ for 1 flip.

We have a probability weight associated w/ each outcome.

1. each weight $w_i \geq 0$

2. $\sum w_i = 1$

e.g. flip 3 times:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

each w/ $w = \frac{1}{8}$

What is the Prob. @ least 1 H occurs? \leftarrow AN **EVENT** ^{lx. \neq}

There is no weight assoc. w/ this
if there were, it would be an

ELEMENTARY EVENT aka
outcome
of
an
EXPERIMENT

An **EVENT** is a SUBSET OF **SAMPLE SPACE**

if E is an EVENT

$P(E)$ = add up the weights in SS that satisfy E

$$= \sum_{x \in E} p(x)$$

= $7/8$ in our example.

A **PROBABILITY FUNCTION** on a SAMPLE SPACE S
satisfies.

1. $P(A) \geq 0 \quad \forall A \subseteq S$
2. IF A and B are disjoint EVENTS then
 $P(A \cup B) = P(A) + P(B)$
3. $P(S) = 1$

if E is an EVENT \bar{E} is **complement** of E , $\bar{E} = S - E$

$$P(\bar{E}) = 1 - P(E)$$

b/c. $P(S) = P(E \cup \bar{E}) = P(E) + P(\bar{E}) = 1$