

CS 305
Design and Analysis of Algorithms

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Today's Topics

- Questions/Comments?
- Last time we covered analysis of MergeSort by creating the recursion tree and adding up the work
- Recurrence Relations
- Logarithms

Recurrence Relations

- For MergeSort
- $\text{Time}(n) = 2 * \text{Time}(n/2) + n$
 - Number of recursive calls is 2
 - The time for one recursive call is $\text{Time}(n/2)$
 - The time in a call is n
- In general for divide and conquer
- $T(n) = a * T(n/b) + f(n)$
 - Number of recursive calls is a
 - n/b is size of list in a recursive call
 - The time in a call is $f(n)$

Recurrence Relations

- In other words, a divide and conquer algorithm that creates a subproblems each a factor of $1/b$ the size of the original problem and takes $f(n)$ amount of time to do the divide and combine steps.
- $T(n) = a * T(n/b) + f(n)$
 - Number of recursive calls is a
 - n/b is size of list in a recursive call
 - The time in a call is $f(n)$

Recurrence Relations

- Let me draw the recursion tree for arbitrary a and b .
- How many leaves?
- Let's see an example with explicit values for a and b .
- How many leaves?
- $T(n) = a * T(n/b) + f(n)$
- Let's prove, by induction, that the number of leaves in a perfect binary tree is 2^d , where $d =$ depth of the tree.

Logarithms

$y = \log_b a$ iff $b^y = a$, $a > 0$, $b > 0$ and $b \neq 1$

- In English: one way to read $\log_b a$ is “what exponent of b results in a ?”
- When $b=2$, $\log_2 a$ is the number of times a can be cut in half (until you hit 1.)
- $\log_2 1024 = ?$ It is the power of 2 that gives you 1024 (or the number of times 1024 be cut in half until you hit 1.)

Logarithms

- Let's show some properties of logarithms on the board.

Logarithms

- Let's go back and look at the a-ary tree and recall there are $a^{(\log_b n)}$ --- that is a to the power $\log_b n$
- Any other way to specify $a^{(\log_b n)}$?

Logarithms

- Let's go back and look at the a-ary tree and recall there are $a^{(\log_b n)}$ --- that is a to the power $\log_b n$
- Any other way to specify $a^{(\log_b n)}$?
- $n^{(\log_b a)}$
- better because it is n to some constant power

Master Method for Recurrences

- Soon we will learn the Master Method which can be used to solve recurrences of the form:
$$T(n) = a * T(n/b) + f(n)$$
- There are 3 cases to consider each based on a relationship of time spent at leaves vs. at the root.
- As an example, and to help us shortly, let's now count the number of nodes in a perfect tree, say a 3-ary tree.