

# CS305: Homework 1

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**Assigned:** Thursday, September 23, 2021

**Due:** Thursday, September 30, 2021

- Do all five of the following problems. **Show your work.** Homework submissions must be emailed to me **as a pdf.**
- You may work with others on the homework, but you **MUST** acknowledge the people you worked with at the top of your homework submission. Do **not** look at the web for solutions to homework problems. Looking for solutions does **not** help your problem solving powers.

## Problems

Note that the book separates things into exercises and problems with exercises appearing at the end of a section and problems (which tend to be longer) appearing at the end of the chapter.

1. For each, either prove (by the definition of  $O$  and  $\Omega$ ) (see p. 44,47,48) or state that it isn't true
  - (a)  $2n^3 + 4n$  is  $O(n^5)$
  - (b)  $2n^3 + 4n$  is  $\Omega(n^3)$
  - (c)  $2n^3 + 4n$  is  $O(n^3)$
2. Answer these with (Yes or No) or (True or False) and a sentence why.
  - (a) Is  $2n^3 - 4n \in \Theta(n^3)$ ?
  - (b) Can you say that  $n^3$  is an asymptotically tight bound for  $2n^3 - 4n$ ?
  - (c) Is  $n^2$  an asymptotic lower bound for  $2n^3 - 4n$ ?
  - (d)  $n^2$  is  $o(n^4)$  Note: little o, not big O
  - (e)  $n^2$  is  $o(n^3)$
  - (f)  $n^2$  is  $o(n^2)$
  - (g)  $n^2$  is  $o(n)$
  - (h)  $4^{lg n}$  is  $\Theta(n^2)$

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3. Problem 4-2 (see p. 107)

4. Prove the following by induction:

$$\text{For all } x \neq 1, \quad \sum_{i=0}^d x^i = \frac{x^{d+1} - 1}{x - 1}$$

5. Determine asymptotically tight bounds for each of the following by the Master Theorem (see p. 94) if possible (and show why the appropriate case holds). If not possible by the Master Theorem, just indicate why.

(a)  $T(n) = 8T(n/2) + 5n^3 + n^2$

(b)  $T(n) = 9T(n/2) + 5n^3 + n^2$

(c)  $T(n) = 7T(n/2) + n^3$

(d)  $T(n) = 6T(n/4) + 3n$