

PROVE THAT A ~~COMPLETE~~ <sup>PERFECT</sup> BINARY TREE ~~OF~~ HAS  $2^d$  NODES AT DEPTH  $d$ .

BASE CASE: When  $d=0$ ,  $2^d = 2^0 = 1$  node  
AND A TREE WITH DEPTH  $0$  ONLY CONTAINS  
A ROOT WHICH IS 1 NODE

$d=1$  base case has been shown.

INDUCTION HYPOTHESIS:

Assume: In a perfect binary tree the number of nodes @  
depth  $d-1$  is  $2^{d-1}$ .

Now we show that @ depth  $d$ , # of nodes is  $2^d$ .

The # of nodes @ depth  $d = 2 * (\# \text{ of nodes @ depth } d-1)$

$$2 * 2^{d-1} = 2^d$$

Q.E.D.