CS 305
Design and Analysis of Algorithms

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Today's Topics

- Questions / Comments?
- Probability
  - Reminder of terms: Sample space, Outcomes, Random Variable, Expected Value of a Random Variable
  - Linearity of Expectation
  - Hiring problem analysis of expected cost to hire
  - Indicator Random Variables
  - Use Harmonic Series (Appendix A.7)
- Use Indicator Random Variables to analyze expected runtime of randomized quicksort

Expected Value and Indicator Random Variables

Expected Value E[.]

X, X1 and X2 are random variables

$X = X1 + X2$

$E[X] = E[X1] + E[X2]$

$E[cX] = cE[X]$, where c is a constant scalar

Indicator Random Variable X_A

S is sample space, A is an Event

$X_A = 1$ if A occurs, 0 if A doesn't occur

$E[X_A] = P(A)$ the probability that A occurs

$$\sum_{i=1}^{n} i \quad \text{if}$$

$$= \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} x^i$$

$$= \frac{x^{n+1} - 1}{x - 1}$$

when $|x| < 1$

$$\sum_{i=0}^{\infty} x^k$$

$$= \frac{1}{1 - x}$$

$$\sum_{i=1}^{n} \frac{1}{i}$$

$$= \ln(n) + O(1)$$